TECHNICAL NOTE

# Approximate equations for the properties of dry steam from a generalization of Callendar's equation

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A generalization of Callendar's equation for dry steam is used to calculate density to ±0.5% and temperature to ±0.8 °C from enthalpy and pressure over the range 50–200 bar and from saturation to 690 °C. Minor rearrangement of the fundamental equation allows pressure to be calculated directly from density and enthalpy.

**Key words**: *steam plant, thermodynamic properties, modelling*

The numerical simulation of the dynamic behaviour of a steam-raising plant makes use of time-integration procedures which require temperature and density to be repeatedly evaluated in terms of enthalpy and pressure at each time step of the solution. A direct use of fundamental formulations for this purpose produces unacceptably long computing times, not least because temperature and pressure are traditionally used as the independent variables, thus entailing a further level of iteration. It is usual, therefore, to seek simpler formulations in which some loss of accuracy and range may be tolerated.

The method of calculation presented here is based on the early work of Callendar[1], who found that very simple expressions relating either enthalpy or internal energy to pressure and density were capable of reproducing the properties of dry steam to good accuracy. Callendar also derived an equation of state containing temperature[2], but not in explicit form and, therefore, it is inconvenient for calculation purposes.

This Note shows firstly, how to improve the accuracy of Callendar's equation, and secondly, how to derive an expression that allows a direct calculation of the dry steam temperature. The work was begun with the objective of finding algebraic expressions to represent the dry-steam tables to good accuracy between 50 and 200 bar and from saturation to 690 °C, which is the maximum temperature in the tables used as the basis for the work[3].

## Method

Motivated by the form of Callendar's enthalpy equation, a set of tables was prepared of the relation between pressure and density for a range of constant enthalpies with a view to plotting these relations. The tables were calculated, spanning the pressure range 10 to 220 bar, by an iterative method using as a basis the Central Electricity Generating Board (CEGB) steam property computer program, the tabular form of which is Ref (3).

Plots of density against pressure for a range of enthalpies are shown in Fig 1; their most marked characteristic is linearity. Fitting the plots by a set of straight lines showed that accuracy was within

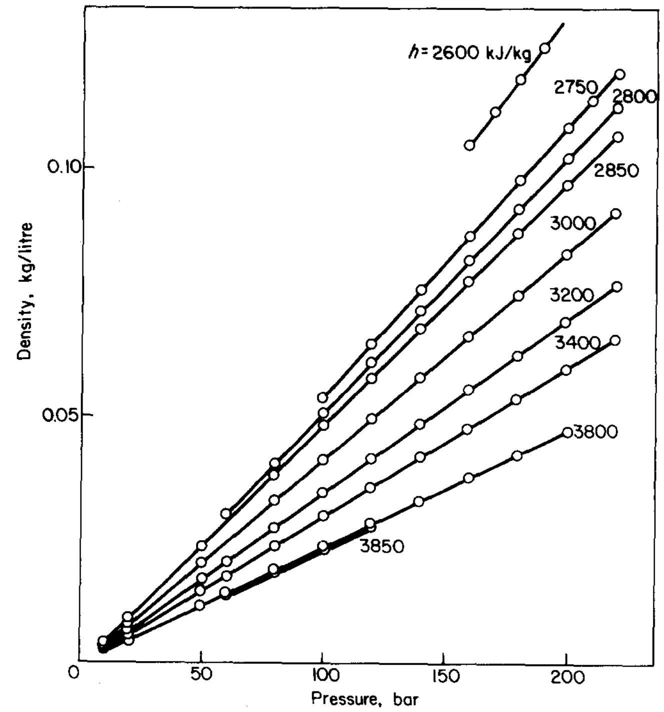

*Fig 1 Plots of density against pressure for a range of enthalpies*

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**Table 1**

| | Enthalpy, kJ/kg | | | | |
|---|---|---|---|---|---|
| Pressure, bar | 2600 | 2750 | 3000 | 3400 | 3800 |
| 50 | | | 1.8,0.15 (0) | 1.7, 0.15 (0) | 2.2, 0.12 (0) |
| 100 | | 0.8, 0.22 (0) | 1.2, 0.22 (+0.18) | 1.5, 0.21 (+0.06) | 3.3, 0.20 (−0.09) |
| 120 | | 0.9, 0.30 (−0.12) | 1.4, 0.20 (+0.10) | 1.9, 0.19 (+0.03) | 4.2, 0.17 (−0.05) |
| 150 | | 1.0, 0.35 (−0.19) | 1.3, 0.26 (−0.01) | 2.9, 0.17 (−0.01) | 3.8, 0.21 (−0.05) |
| 170 | 0.6, 0.38 (0) | 1.2, 0.38 (−0.14) | 1.4, 0.32 (−0.04) | 2.9, 0.20 (−0.02) | 4.2, 0.21 (−0.03) |
| 200 | 0.6, 0.39 (0) | 0.9, 0.30 (0) | 1.4, 0.24 (0) | 2.8, 0.18 (0) | 4.1, 0.24 (0) |
| 220 | 0.6, 0.38 (+0.18) | 1.1, 0.38 (+0.15) | 1.3, 0.27 (+0.08) | 2.7, 0.21 (+0.04) | |

*The tabulated quantities are skeleton table tolerances on temperature and density in °C and per cent, respectively. The quantities in brackets are percentage errors of linear approximations to the density plots of Fig 1*

the tolerances of the International Skeleton Table[4] in the pressure range 50–220 bar, and Table 1 shows the percentage error for various cases. However, significant departures from linearity appear at <50 bar, particularly at lower values of enthalpy. For example, at a value of 2850 kJ/kg, the linear fit is below the true density by 1.3% at 30 bar and by 10% at 10 bar.

Within the restricted pressure range, and with a small additional loss of accuracy towards 50 bar, it was further established that the pressure–density graphs could be reduced to a pencil of lines whose projections intersect at a common point close to the origin as embodied in Eq (1):

$$\phi(h)[\rho - \rho_R] = P - P_R \tag{1}$$

The slope $\phi$ is a function of enthalpy which deviates only by a few per cent from linearity. However, it is crucial to reproduce it as accurately as possible and Eq (1.1) is adequate in this respect:

$$\phi(h) = 2992.6 + 2.3485(h - h_R) - 0.26202 \times 10^{-6}(h - h_R)^3 \tag{1.1}$$

where $h_R = 3263.2$ kJ/kg; $P_R = 2.75$ bar; $\rho_R = 0.65 \times 10^{-3}$ kg/litre.

Eqs (1) and (1.1) relating enthalpy to pressure and density are the basis of this work.

## To calculate temperature

Callendar found two independent expressions containing temperature which were functions of entropy alone. Therefore, one of these expressions could be regarded as a function of the other resulting in an equation of state relating pressure, volume and temperature. However, it has been noted previously that temperature cannot be found by direct calculation from this equation, but it is possible to modify the method so that only one of the entropy-dependent expressions contains the temperature.

According to the definition of enthalpy, an isentropic change is given by Eq (2) where the factor 0.1 is necessitated by the steam-table units:

$$\delta h = 0.1 v\, \delta P \tag{2}$$

The density can be eliminated using Eq (1) to give:

$$\frac{d(P - P_R)}{dh} - 10(P - P_R)\phi^{-1} = 10\,\rho_R \tag{3}$$

This is a first-order linear differential equation for an expression connecting pressure and enthalpy at constant entropy. The integrating factor is given in Eq (3.1) and the corresponding solution in Eq (3.2):

$$I = \exp - 10 \int \phi^{-1}\, dh \tag{3.1}$$

$$I(P - P_R) - 10\rho_R \int I\, dh = E(S) \tag{3.2}$$

$E$ is the constant of integration which must depend upon entropy. The integrating factor together with its integral are evaluated in the Appendix, and the results are given in Eqs (3.3) and (3.4), respectively. As the integral of $I$ appears only when multiplied by the small reference density, it was found sufficient to use the approximation:

$$I = (6760.2 - h)^{2.0649}\phi^{-0.68829} \times \exp(-15.8058 \arctan (2.18886 \times 10^{-3} h - 3.31538)) \tag{3.3}$$

$$\int I\, dh \simeq -0.1\phi I(1 - 0.1\phi'_m)^{-1} \tag{3.4}$$

**Notation**

(*Units* as in the *SI Steam Tables*)

| | |
|---|---|
| $P$ | Pressure |
| $S$ | Entropy |
| $h$ | Enthalpy |
| $T$ | Temperature |
| $v$ | Specific volume |
| $\rho$ | Density |
| $\phi$ | Cubic polynomial of enthalpy |
| $\phi'_m$ | Mean gradient of $\phi$ |
| $I$ | Integral defined in the text |
| $E$ | Constant of integration dependent upon entropy |
| $x$ | Deviation of enthalpy from the reference level |
| $x_R$ | Real root of $\phi$ |
| $Y$ | Defined by Eq. (7.1) |
| $Z$ | Defined by Eq. (7.2) |

Suffix R denotes a reference level

In this expression, $\phi'_m$ is the mean slope of $\phi$, and as $\phi$ does not depart significantly from a straight line over much of its range, the mean slope is taken to be 2.28. Using Eq (3.4), therefore, and eliminating pressure by means of Eq (1), an expression containing density and enthalpy is obtained which depends only upon entropy:

$$(\rho + \rho_{R2})\phi I = E(S) \tag{4}$$

where

$$\rho_{R2} = \rho_R\left(\frac{10}{\phi_m} - 1\right)^{-1}$$
$$= 0.192 \times 10^{-3}$$

It is now necessary to find a second expression which depends only upon entropy, but which contains the temperature. As temperature is the derivative of enthalpy with respect to entropy at constant pressure, it is possible to do this by differentiating Eq (3.2), accordingly:

$$(P - P_R)\left(\frac{dI}{dh}\right)\left(\frac{\partial h}{\partial S}\right)_P - 10\,\rho_R I\left(\frac{\partial h}{\partial S}\right)_P = \frac{dE}{dS}$$

By differentiating Eq (3.1) also, it follows that:

$$\frac{dI}{dh} = -10 I \phi^{-1}$$

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Making the substitutions, and eliminating pressure using Eq (1) gives the required isentropic expression containing the temperature:

$$I\rho T = -0.1\frac{dE}{dS} \tag{5}$$

Eqs (4) and (5) taken together are a parametric form of a univariate function connecting $I\rho T$ with $(\rho + \rho_{R2})\phi I$ and, once the function has been determined, it is a simple matter to calculate temperature from any given enthalpy and pressure or enthalpy and density.

The function was obtained by calculating values of $I\rho T$ and $(\rho + \rho_{R2})\phi I$ for a set of pressures and entropies spanning the appropriate range of the dry-steam tables. It was noted that these values were virtually independent of pressure as predicted by the theory and, therefore, that $I\rho T$ could be cross-plotted as a function of $(\rho + \rho_{R2})\phi I$. In practice, the variables $Y, Z$, defined in Eq (7) were used, which had the effect of greatly reducing the numerical range of the variables and also of rendering the function closely linear (Eq (8)):

$$Y = -\ln(I\rho T) \tag{6}$$

$$Z = -\ln(\phi I(\rho + \rho_{R2})) \tag{7}$$

$$Y = 1.1231Z + 0.64 + e(Z) \tag{8}$$

The correction $e(Z)$ is defined by the set of points in Table 2, and intermediate values may be found by spline interpolation.

The procedure may be summarized as follows. Given pressure and enthalpy, then $\rho, \phi, I$ may be calculated from Eqs (1), (1.1) and (3.3). $Z$ may then be found from Eq (7) and, therefore, also $Y$ from Eq (8). The temperature is then obtained from Eq (6) and the complete calculation is accomplished in a few lines of computer code.

**Table 2 Tabulation of the correction *e*(*Z*)**

| $Z$ | $e(Z)$ |
|---|---|
| −0.2974 | 0.00000 |
| 0.0157 | 0.01629 |
| 0.5683 | 0.04725 |
| 0.9671 | 0.07073 |
| 1.4533 | 0.10170 |
| 1.7770 | 0.11853 |
| 2.4822 | 0.15749 |
| 3.1120 | 0.18317 |
| 3.6827 | 0.19632 |
| 3.9537 | 0.19756 |
| 4.2159 | 0.19527 |
| 4.7189 | 0.18459 |
| 5.2002 | 0.16343 |
| 5.6657 | 0.13394 |
| 6.1190 | 0.10084 |
| 6.4350 | 0.072657 |
| 6.5640 | 0.063945 |
| 7.0040 | 0.022763 |
| 7.2240 | 0.000315 |
| 7.2973 | −0.002924 |
| 7.5662 | −0.031233 |
| 7.7238 | −0.045180 |

## Accuracy

The fundamental formula of Eq (1), and the temperature formula of Eq (6) have been tested over the range 50–200 bar and from saturation to 690 °C against data from the CEGB steam tables[3]. The errors in calculated density range from 0.3% high to 0.6% low and these can be directly attributed to similar errors in the algebraic fit to the slope $\phi(h)$. By an *ad hoc* change in the reference pressure from 2.75 to 2.65 bar, the density error becomes evenly balanced to ±0.5%. All of the temperature calculations gave results correct to ±0.8 °C and the tolerances listed in Table 1 were never exceeded. The errors on the saturation line were at a maximum at a pressure of 150 bar where the density and temperature were 0.5% and 0.55 °C low, respectively. The temperature tolerances were estimated by dividing the enthalpy tolerance from the skeleton tables by an approximation to the local specific heat.

In view of the known decrease in accuracy at <50 bar of the assumed linear relations between density and pressure, it is of interest to examine the accuracy of the derived formulae at lower pressures. The maximum errors found at 35 bar pressure were 1% in density and 2.5 °C in temperature, while at 30 bar, these increased to 2% and 6 °C, respectively. On the saturation line, the errors remain < 1% and < 1 °C at 35 bar, increasing to 2% error in density and 1.5 °C error in temperature at 30 bar pressure.

## Discussion

The fundamental assumption of linearity is invalid at pressures ≪ 50 bar, undoubtedly due to the fact that the isenthalpic lines have ultimately to bend to pass through the origin, so that they may approximate to an ideal gas at very low pressures. A more general analytical treatment would therefore be necessary to represent the behaviour of dry steam at lower pressures. However, the present formulae may retain some utility down to 35 bar, although probably not to <30 bar pressure.

## Conclusions

Simple algebraic equations can provide dry-steam density correct to ±0.5% and temperature to ±0.8 °C with respect to steam table values[3] over the range 50 to 200 bar and from saturation to 690 °C. Alternatively, pressure can be the dependent variable calculated from density.

The method is a development of that of Callendar and is based upon the observation that, over the given pressure range, density at constant enthalpy is a linear function of pressure within the tolerance of the skeleton steam tables. The novelty is the finding that the slopes of the pressure–density lines are not linear in enthalpy and also that an equation explicit in temperature can be found.

The new equations have been derived only to facilitate engineering calculations. No attempt has been made to re-assess the experimental data upon which the steam tables are based.

## Acknowledgement

Thanks are due to Mr G. H. Williams of the Central Electricity Research Laboratories for preparing the tabulations of density against pressure at constant enthalpy, and also at constant entropy, upon which this work is based.

The work was carried out at the Central Electricity Research Laboratories and is published by permission of the Central Electricity Generating Board.

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## Appendix

### To find $I$ as defined by Eq (3.1)

The function $\phi$ is defined by Eq (1.1). On substituting $x$ for $h-h_R$, $\phi$ can be factorized into one real root and a complex pair:

$$\phi(x)=(x_R-x)(A_0+A_1x+A_2x^2)$$

where

$$A_0=0.855756 \qquad (1A)$$

$$A_1=0.916283\times10^{-3}$$

$$A_2=0.262017\times10^{-6}$$

$$x_R=3497.030$$

The following partial fraction expansion can therefore be made:

$$\frac{1}{\phi(x)}=\frac{B}{x_R-x}+\frac{Cx+D}{A_0+A_1x+A_2x^2}$$

where

$$B=0.137660$$

$$C=0.036069\times10^{-6} \qquad (2A)$$

$$D=0.252270\times10^{-3}$$

The first component $I_1$ of the integral $I$ is clearly logarithmic:

$$I_1=\exp-10\int\frac{B\,\mathrm{d}h}{x_R-x}$$

$$=\exp(10B\ln(x_R-x))$$

Therefore

$$I_1=(x_R-x)^{1.3766} \qquad (3A)$$

The second component $I_2$ is most directly found from a table of integrals. One tabulated result shows how to expand the integral as follows:

$$\int\frac{Cx+D\,\mathrm{d}x}{Q}=C\left(\frac{1}{2A_2}\ln Q-\frac{A_1}{2A_2}\int\frac{\mathrm{d}x}{Q}\right)+D\int\frac{\mathrm{d}x}{Q}$$

where $Q$ denotes the quadratic expression

$$=\frac{C}{2A_2}\ln Q+\left(D-\frac{A_1C}{2A_2}\right)\frac{2}{F}\arctan\left(\frac{2A_2x+A_1}{F}\right)$$

by further reference to the table of integrals[5]. Component $I_2$ is, therefore, as follows:

$$I_2=\exp-10\int\frac{Cx+D}{Q}\,\mathrm{d}x$$

$$=Q^{-10G}\exp\left(-10H\arctan\left(\frac{2A_2x+A_1}{F}\right)\right)$$

$$=\left(\frac{\phi}{x_R-x}\right)^{-10G}\left(\exp-10H\arctan\left(\frac{2A_2x+A_1}{F}\right)\right)$$

after substituting for $Q$ in terms of $\phi$ using Eq (1A).

Multiplying $I_1$ by $I_2$, therefore, gives the required expression for $I$:

$$I=(x_R-x)^{2.0649}\phi^{-0.68829}$$

$$\times\exp(-15.80576\arctan(2.18886\times10^{-3}x$$

$$+3.82726)) \qquad (4A)$$

**To find an approximate expression for $\int I\,dh$**

Differentiating Eq (3.1) gives:

$$0.1\phi\frac{dI}{dh}+I=0$$

$$\therefore\quad \frac{d(0.1\phi I)}{dh}-I\frac{d(0.1\phi)}{dh}+I=0$$

$$\therefore\quad \frac{d(0.1\phi I)}{dh}=(0.1\phi'-1)I$$

$$\therefore\quad 0.1\phi I=\int(0.1\phi'-I)I\,dh$$

$$=(0.1\phi'_m-1)\int I\,dh$$

using the first mean value theorem and where $\phi'_m$ is a value of $\phi'$ within the range of integration. If some approximate value is assumed for $\phi'_m$ it follows that:

$$\int I\,dh\simeq 0.1\phi I/(0.1\phi'_m-1) \qquad (5A)$$

# BOOK REVIEWS

## Convective Boiling and Condensation

J. G. Collier

The first edition of John Collier's book on convective boiling and condensation has been the recognised authoritative text in this area since its publication in 1972. This, the second edition, retains the general format of the original edition, and the changes in the text have not resulted in any significant increase in length; the original 421 pages has expanded to 435 pages.

There are twelve chapters; following an introduction there are two chapters on two-phase flow, six chapters on boiling, a chapter on condensation, and finally two chapters on aspects common to both boiling and condensation. The greater part of the introductory chapter concerns the flow patterns that occur in two-phase pipe flow. The following two chapters on two-phase flow are primarily concerned with the prediction of pressure drop in pipes and pipe fittings. Most established methods are presented; the new edition covers some of the more important developments since 1972.

Almost half of the book (6 or 12 chapters) concerns boiling. It is on this topic that I believe this text is without equal, both in relation to the technical coverage and the text's 'readability'. The first of these chapters, An Introduction to Convective Boiling, covers basic aspects of the boiling process; bubble growth, nucleation sites, bubble dynamics, pool boiling, heat transfer regimes, boiling maps and so forth.

The next chapter, 'Subcooled Boiling Heat Transfer', has changed little since the first edition; the latest reference is 1968. Similarly the following chapter, 'Void Fraction and Pressure Drop in Subcooled Boiling', has only been modestly amended, although the next chapter, 'Saturated Boiling Heat Transfer', has been more significantly updated.

There are two chapters on critical heat flux; the first of these concerns vertical uniformly heated tubes (the simplest case) and the second, more complex geometries. The latter chapter has been considerably updated.

The chapter on condensation covers most aspects of this topic of interest to the well informed condenser designer, starting from droplet nucleation, through accommodation coefficients, Nusselt's laminar film theories and so forth to dropwise condensation. The section on 'Pressure Drop in Condenser Tube Banks' seemed too superficial, but I write with a specialist interest in that area.

The final two chapters are concerned with conditions influencing the performance of boiling and condensing systems, and multicomponent boiling and condensation. This latter chapter replaces the previous final chapter, 'Two-component Two-phase Heat Transfer'. The new chapter will be of more industrial interest.

Each chapter concludes with a number of problems (the solutions are available from the author) and eight of the chapters have a numerical example. SI units are used throughout. The general presentation of the text is first class, with bold type and clearly presented tables and figures.

The text, the Preface states, is intended for the design engineer in the power plant and process industries. It will also be of interest and use to university staff and research workers concerned with processes involving change of phase. It is recommended.

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